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Computation of intraluminal impedance

Awad Al-Zaben and V Chandrasekar

Electrical and Computer Engineering, Colorado State University, Fort Collins, CO 80524, USA

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Abstract
The measurement of the electrical impedance inside the esophagus provides information about its status, and is being explored in the study of the gastroesophageal reflux. This paper presents theoretical computation of impedance inside the esophagus. The results of the numerical solution for a simple geometry are compared against the solution formulated from the Green's function approach. The effect of the electrode configuration on the resulting impedance is studied as an application of the methodology developed in this paper. The results of this paper will be useful in the design of an intraluminal impedance catheter as well as in the interpretation of the resulting impedance signals

Keywords: intraluminal impedance, reflux, finite element, Green's function

1. Introduction

The measurement of electrical impedance inside the gastrointestinal tract (GIT) is commonly referred to as multichannel intraluminal impedance (MII). The multichannel intraluminal impedance (Al-Zaben and Chandrasekar 2003, Silny 1991, Silny *et al* 1993, Skopnik *et al* 1996, Srinivasan *et al* 2001, Trachterna *et al* 1999) system uses an insulating catheter with a number of current carrying metallic ring electrodes placed inside the esophagus as shown in figure 1. The differential voltage between pairs of electrodes is used to measure the average electrical impedance of the volume of conductor around the electrodes. A pair of electrodes is typically referred to as a channel. The impedance changes are used to study the gastroesophageal reflux patterns and the esophagus clearance (Shay *et al* 2002, Srinivasan *et al* 2001).

The measured impedance depends on many factors including electrode configuration (i.e. electrode length, inter-electrode spacing), esophagus conductivity and the conductivities of the materials inside the esophagus. However, by choosing a suitable configuration of the catheter, the events occurring inside the esophagus such as swallow or gastroesophageal reflux can be clearly observed by monitoring the impedance values. The interpretation of the impedance observations is not a trivial task. The impedance traces can vary from patient to patient as

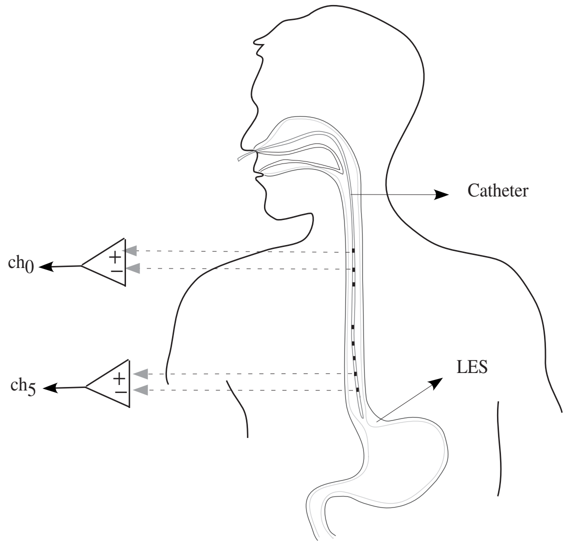

Figure 1. Catheter configuration and placement inside the esophagus.

well as within the same patient during the study period, in addition to the variability due to esophageal contents. Therefore in order to study the impedance characteristics due to various esophageal conditions as well as interpret rare impedance observations, it is important to develop theoretical computation of the impedance measurement. Simple models such as modeling the esophagus as a perfect cylinder may not be sufficient to model and interpret various impedance changes in practice. For example, inhomogeneity in bolus conductivities, esophagus wall conductivity, catheter configuration, all these effects can be modeled accurately only in a numerical solution. This paper presents numerical computation of the impedance observed in the esophagus. The paper is organized as follows. Section 2 presents a simple model for the catheter inside the esophagus and presents the solution via the Green's function approach. Section 3 presents the numerical computation of the impedance using the finite element (FE) approach. This section also presents the underlying equations to set up the finite element model. The impedance due to a bolus at a specific location in the esophagus with realistic geometries, as opposed to a simple cylindrical model, is presented here. An important application of this paper is the ability to study the impact of electrode locations in the probe design. Examples of the impact of electrode location on the impedance are presented in section 4. The main results of this paper are summarized in section 5.

2. Simple model of the esophagus

One of the early attempts to compute the impedance measured by a catheter inside the esophagus was presented in Silny (1991). This computation was based on a simplified application of the method of images in a cylindrical coordinate system. The following presents the theoretical computation of impedance in the cylindrical coordinate system to a higher level of accuracy. As a first-order approximation the catheter inside the esophagus can be modeled in the cylindrical coordinate system as described below.

Figure 2 shows a simple model of the catheter inside the esophagus, where the esophagus is modeled as a dielectric cylinder placed in a medium of uniform conductivity extending to infinity. The probe is also cylindrical, located at the center of the coordinate system, where the

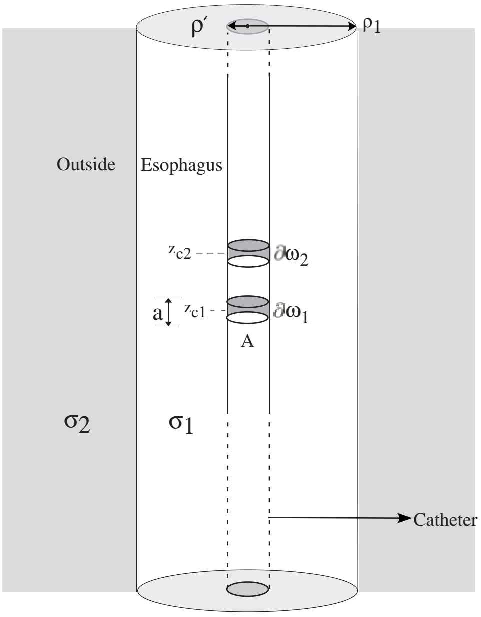

Figure 2. A simple cylindrical model of two electrodes inside the esophagus.

probe length is aligned along the z-axis. The MII probes use low frequency current to monitor the esophagus. Therefore the governing equation for the potential is given by

$$\nabla \cdot (-\sigma(\mathbf{r})\nabla\psi(\mathbf{r})) = J(\mathbf{r}') \tag{1}$$

where $J(\mathbf{r}')$ is the current source at $\mathbf{r}'$ (Kleinermann *et al* 1999).

The solution for the potential can be written in terms of the Green's function in the integral form as

$$\psi(\rho, \theta, z) = \int_A G(\mathbf{r}, \mathbf{r}')J(\mathbf{r})\,\mathrm{d}A \tag{2}$$

where A is the electrode surface area as shown in figure 2, and $G(\mathbf{r}, \mathbf{r}')$ is the Green's function that vanishes at ∞ and satisfies the following equation:

$$\nabla^2 G(\mathbf{r}, \mathbf{r}') = -\delta(\mathbf{r} - \mathbf{r}') = -\frac{\delta(\rho - \rho')}{\rho}\delta(\theta - \theta')\delta(z - z'). \tag{3}$$

It can be shown for thin electrodes that can be modeled as a line source of length a and $\rho' \approx 0$ that the potential within the esophagus ($0 < \rho \leqslant \rho_1$) is given by (Smythe 1967)

$$\psi(\rho, \theta, z) = \frac{1}{2\pi^2}\sum_{n=1}^{N}\frac{I_n}{a\sigma_1}\int_{z_c-a/2}^{z_c+a/2}\int_0^{\infty}(K_0(\alpha\rho) + \kappa I_0(\alpha\rho))\cos(\alpha(z - z'))\,\mathrm{d}\alpha\,\mathrm{d}z' \tag{4}$$

where N is the number of electrodes, I_n is the current in the nth electrode, I_0 is the modified Bessel function of first kind and K_0 is the modified Bessel function of second kind, σ_1 is layer 1 (bolus) conductivity, σ_2 is layer 2 (esophagus and volume conductor) conductivity, and

$$\kappa = \frac{(\sigma_1 - \sigma_2)K_0(\alpha\rho_1)K_1(\alpha\rho_1)}{\sigma_1 K_0(\alpha\rho_1)I_1(\alpha\rho_1) + \sigma_2 I_0(\alpha\rho_1)K_1(\alpha\rho_1)}. \tag{5}$$

Figure 3 shows the potential obtained from equations (4) and (5) as a function of the distance from the center (ρ) at two Z values coinciding with those of the electrode locations. The boundary of the esophagus is assumed at $\rho_1 = 0.5$ cm.

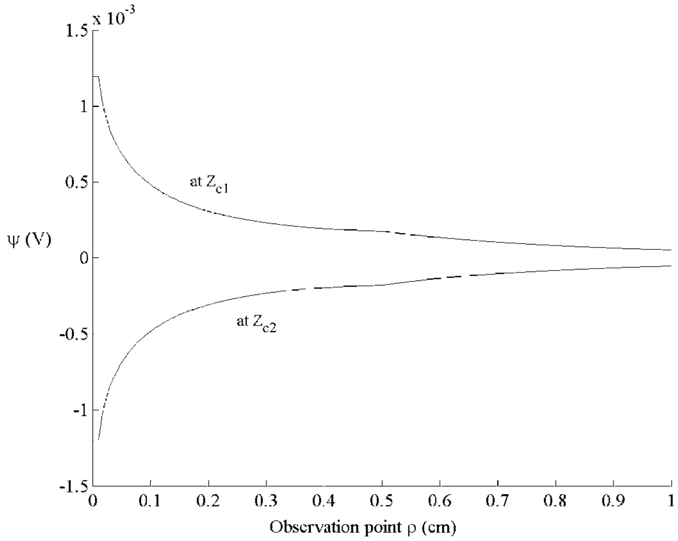

Figure 3. Potential obtained from a pair of electrodes as a function of the distance from the center. The two curves show the potentials for two Z values coinciding with the electrode locations. The boundary of the esophagus is assumed at $\rho_1 = 0.5$ cm.

The impedance as a function of the boundary thickness (ρ_1) for conductivities inside and outside the esophagus of $\sigma_1 = 5 \times 10^{-3}$ S cm^{-1} and $\sigma_2 = 1 \times 10^{-3}$ S cm^{-1}, respectively, is shown in figure 4(a) and for conductivities $\sigma_1 = 15 \times 10^{-3}$ S cm^{-1} and $\sigma_2 = 3 \times 10^{-3}$ S cm^{-1} is shown in figure 4(b).

3. Finite element computation

The governing equation for the potential given by equation (1) can be written as a Laplace equation with boundary conditions at the electrodes as

$$\nabla \cdot (-\sigma(x, y, z)\nabla\psi(x, y, z)) = 0 \tag{6}$$

subject to the boundary conditions (Cheng *et al* 1989)

$$\begin{cases} -\sigma(x, y, z)\frac{\partial\psi}{\partial n} = \mathbf{J}_1 & \text{at} \quad \partial\Omega_1 \\ -\sigma(x, y, z)\frac{\partial\psi}{\partial n} = -\mathbf{J}_1 & \text{at} \quad \partial\Omega_2 \\ \psi(x, y, z) = 0 & \text{at} \quad \partial\Omega_3 \end{cases} \tag{7}$$

where $\partial\Omega_1$ is the surface of the first electrode, $\partial\Omega_2$ is the surface of the second electrode, $\partial\Omega_3$ is the surface of the outside boundary and $\mathbf{J}_1$ is the current density.

The finite element method (Segerlind 1984, Whiteman 1963) requires a piecewise approximation of the governing equation. The strong form of the Laplace equation cannot be used directly because it requires that the second derivative of the solution exists. An alternate form of the solution can be derived by the use of the variational calculus (Morse and Feshbach 1953). The finite element technique is very useful to model the realistic geometry of the

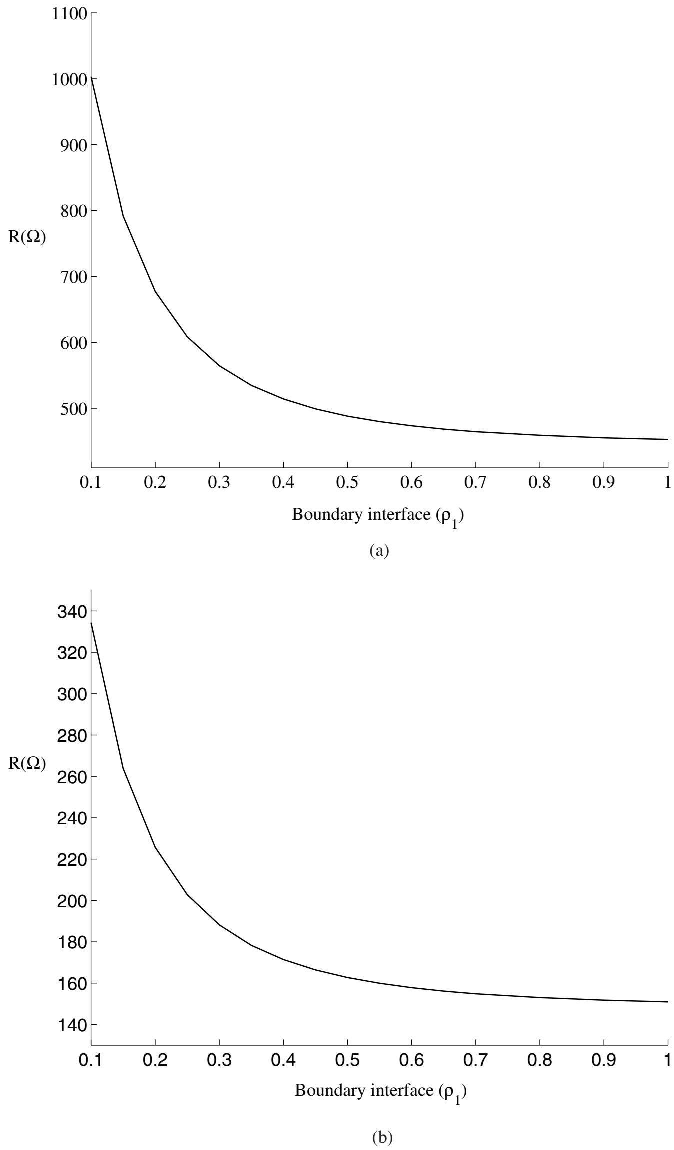

Figure 4. Impedance as a function of the bolus thickness: (a) for conductivities inside and outside the esophagus of $\sigma_1 = 5 \times 10^{-3}$ S cm^{-1} and $\sigma_2 = 1 \times 10^{-3}$ S cm^{-1}, respectively; (b) for conductivities $\sigma_1 = 15 \times 10^{-3}$ S cm^{-1} and $\sigma_2 = 3 \times 10^{-3}$ S cm^{-1}.

catheter inside the esophagus for the following reasons:

- the esophagus is not a perfect cylinder;
- the bolus shape is not a perfect cylinder;
- inhomogeneity in the conductivities;
- finite dimensions of the catheter;
- finite dimensions of the electrodes and finite conductivity.

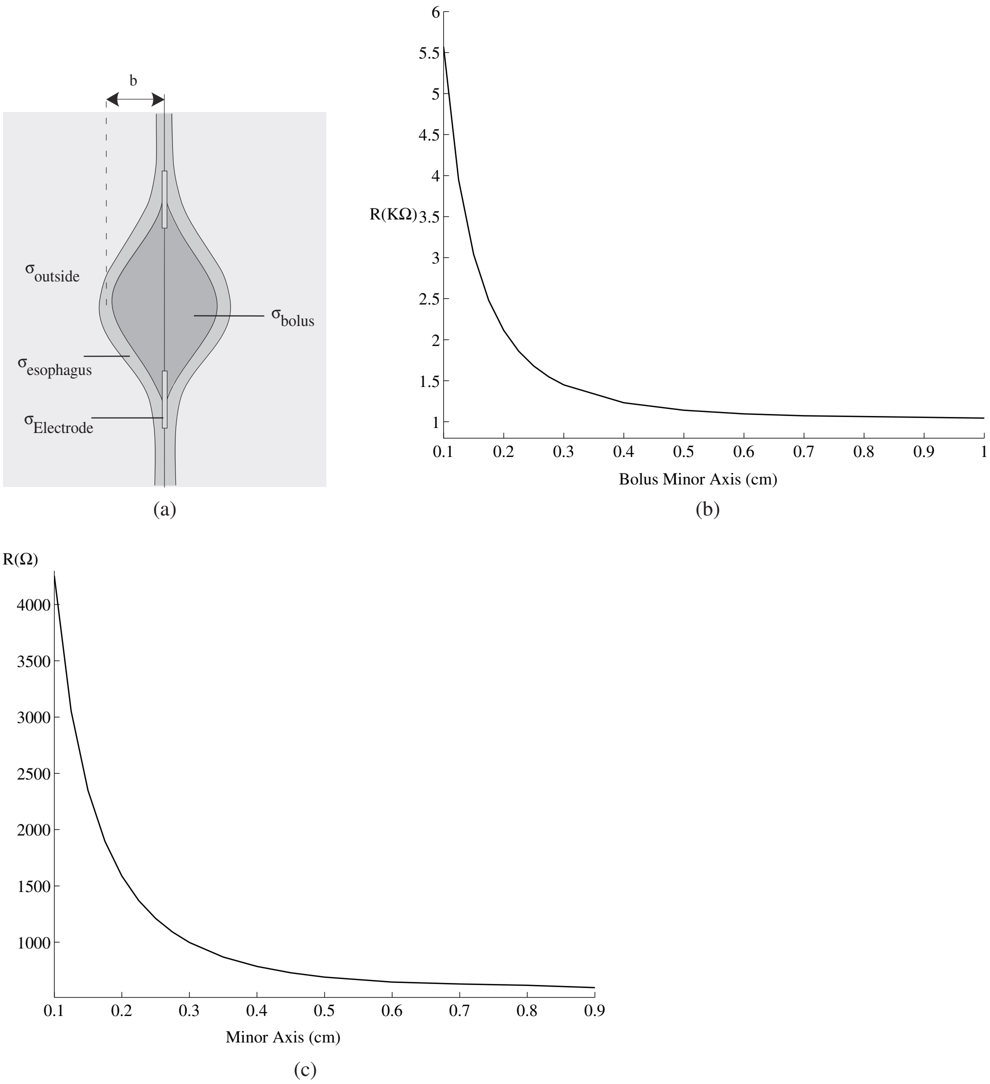

Figure 5. An example of one of the cases of the bolus around the catheter (cylindrical coordinate); electrode thickness is 0.008 cm. Electrodes are of steel material carrying a current of 4 μA, esophagus thickness is 0.1 cm with a conductivity of 5×10^{-6} S cm^{-1}, the outside layer conductivity is 1×10^{-3} S cm^{-1} and the bolus conductivity is 5×10^{-3} S cm^{-1}. (a) Model of bolus. (b) Impedance as a function of the bolus minor axis keeping the major axis fixed. (c) Impedance as a function of the bolus minor axis keeping the bolus volume fixed.

The ability to model realistic and non-ideal observation conditions is demonstrated in the following. Let the esophagus wall thickness be 0.1 cm and a bolus is of spheroidal shape. The axis of the spheroid is aligned along the electrodes. Spheroidal shapes were chosen because they can represent a wide variety of bolus shapes depending on the axis ratio. The minor axis of the bolus varies from 0.1 cm to 1 cm, while the major axis is kept constant at 1 cm. The model configuration is shown in figure 5(a). Figure 5(b) shows the variability of impedance as a function of the minor axis of the spheroid (or the thickness of the bolus). Figure 5(c) shows the impedance as a function of the bolus minor axis while keeping the volume of the bolus fixed. Figures 5(b) and (c) show that as the bolus thickness increases the impedance decreases.

3.1. Line sources

The difference between the finite element solution and the analytical solution for the two simple cylindrical models considered in the previous section was negligible (less than 0.025%). It can be seen that the finite element solution provides accurate results of the impedance. In the following section the finite element method is used to investigate the various factors that affect the impedance using a realistic model for the bolus volume and a catheter with electrodes that are of steel material and have finite thickness.

4. Electrode location

4.1. Inter-electrode spacing

Inter-electrode spacing is defined as the space between the electrodes. This is an important parameter that determines the number of channels used in the catheter and the minimum bolus length that can be detected. The catheter is assumed to have a radius of 0.05 cm and 4 mm electrode length. The esophagus is assumed to be of 0.3 cm thickness and has a conductivity of 1.27×10^{-3} S cm^{-1}, the bolus material is assumed to have a conductivity of 15.5×10^{-3} S cm^{-1}. The outer layer has a conductivity of 9×10^{-4} S cm^{-1}. Figure 6 shows the impedance obtained from the FE solution for two cases: case 1 without any bolus material between the two electrodes is shown in figure 6(a) and case 2 with bolus material of radius 0.5 cm is shown in figure 6(b). Figures 6(a) and (b) show that the impedance increases with increasing electrode spacing and it can be seen that the change in impedance is large between the conditions of no bolus material and the presence of bolus. The exact numerical values depend on the esophagus and the various conductivities in the model. The ratio of the impedance between the absence and presence of bolus is plotted as a function of the electrode spacing for various esophagus conductivities and is shown in figure 6(c). This family of curves can be used as rudimentary design curves where the electrode spacing can be chosen to provide a certain level of impedance variation with bolus.

4.2. Catheter radius

The following analysis evaluates the effect of accounting for finite catheter size, since the catheter has finite radius. The catheter radius is studied for an example where the electrode length is 4 mm and the electrode spacing 2 cm. The two cases of absence and presence of bolus were studied. Figure 7(a) shows the impedance obtained for the no bolus case whereas figure 7(b) shows the impedance when the bolus is present. In both figures 7(a) and (b) the esophagus radius is 0.5 cm. It can be seen from the results of figure 7 that the impedance changes with the catheter radius. An important design factor of the catheter is that it must not impair the esophagus function (Silny 1991), therefore the selection of the catheter radius must be made as small as possible while ensuring sufficient impedance variation can be observed. The ideal solution using a line source model and the Green's function approach would yield an impedance close to 160 Ω independent of the catheter radius. Thus figure 7 shows the utility of the computation developed in this paper. The ratio of the impedance between the absence and presence of bolus is plotted as a function of the catheter radius for various esophagus conductivities and is shown in figure 7(c). It can be clearly seen from

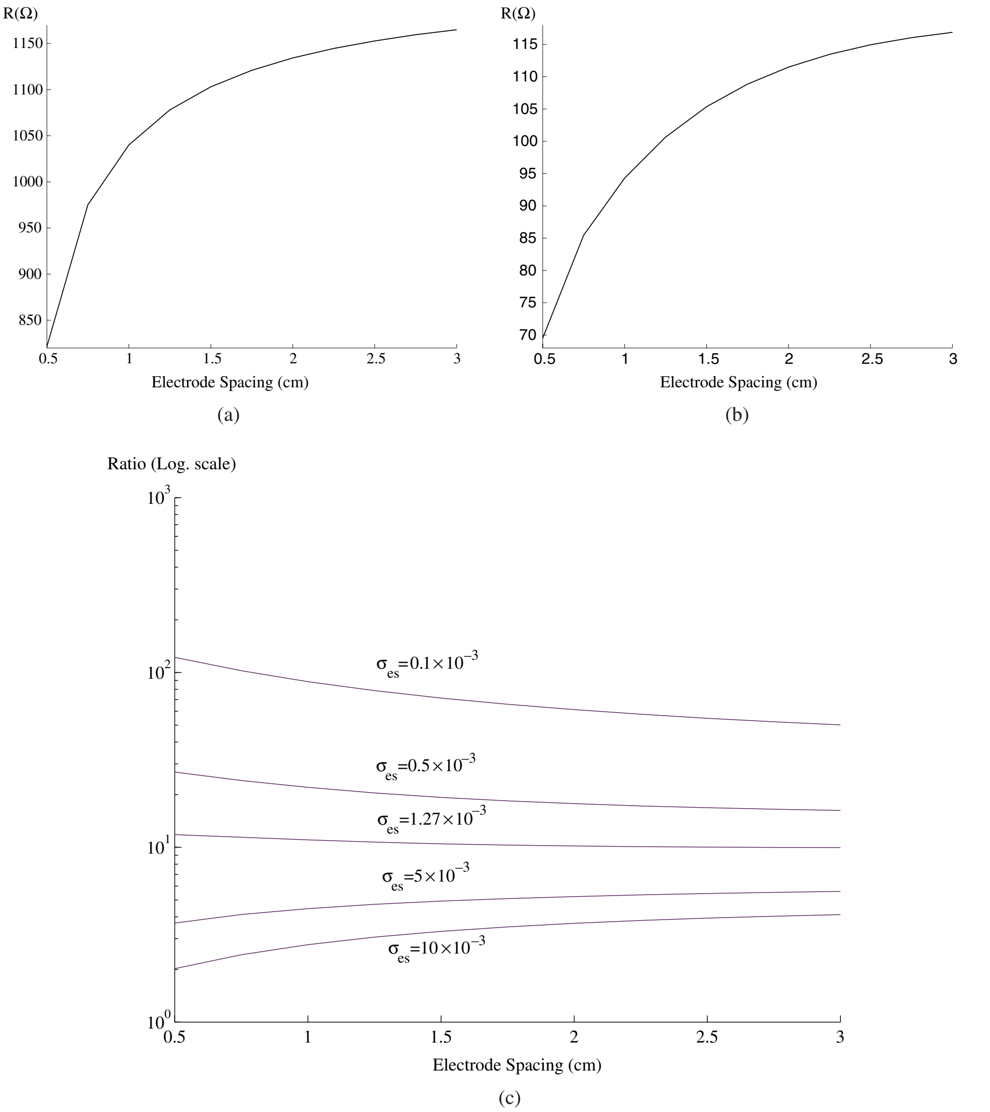

Figure 6. Impedance as a function of inter-electrode spacing: (a) case of no bolus material, esophagus thickness is 0.3 cm and electrode thickness = 0.008 cm; (b) case with bolus material of conductivity $\sigma = 15.5 \times 10^{-3}$ and radius of 0.5 cm, esophagus thickness is 0.3 cm and electrode thickness = 0.008 cm; (c) ratio of the impedance between the absence and presence of bolus as a function of the electrode spacing for various esophagus conductivities.

figure 7(c) that the sensitivity to catheter size is not very high and therefore one can design the smallest possible catheter.

5. Discussion

This paper presents analytical and numerical solutions for the MII problem. The analytical solution for MII impedance is provided using a simple cylindrical model. However the

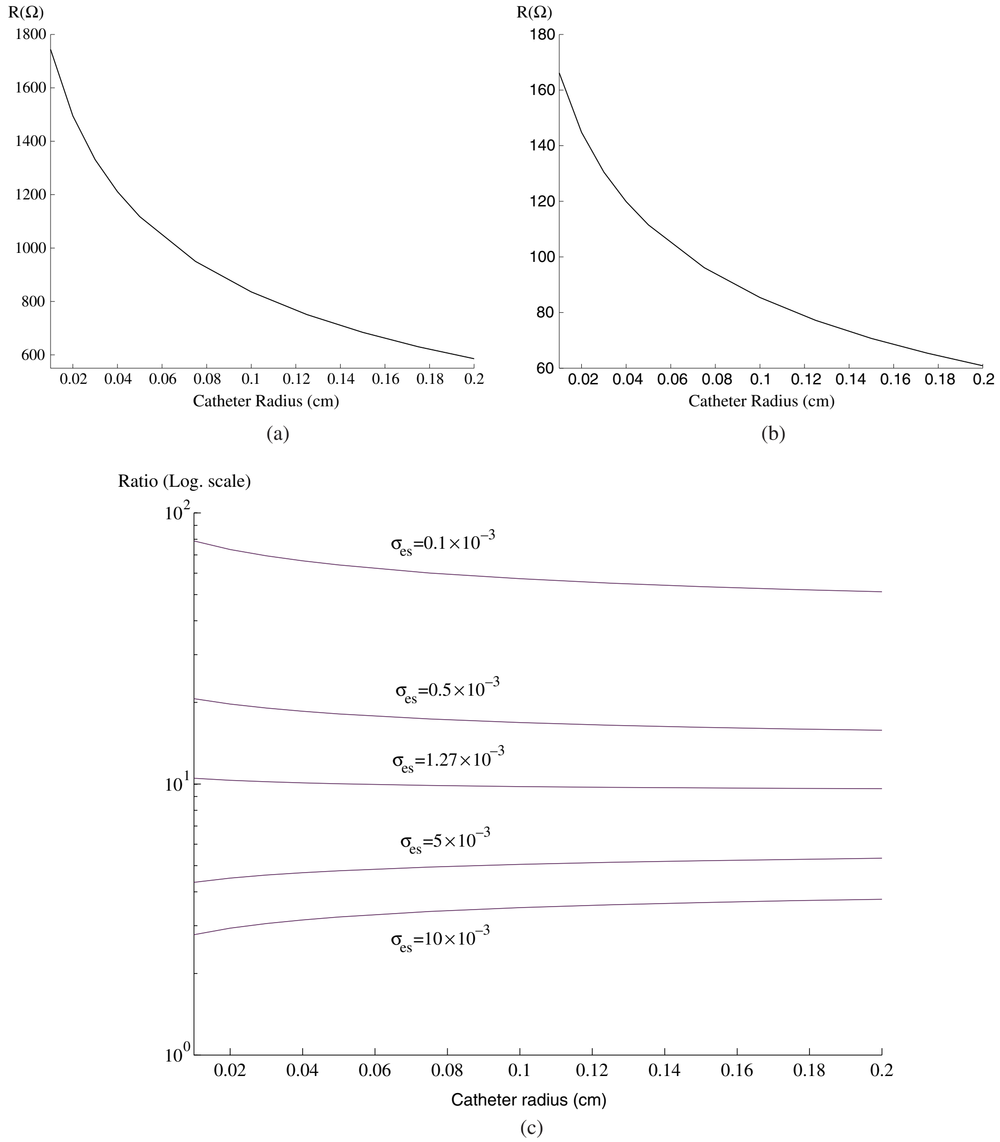

Figure 7. Impedance as a function of catheter radius: (a) case of no bolus material, esophagus thinkness is 0.3 cm and electrode thickness = 0.008 cm; (b) case with bolus material of conductivity $\sigma = 15.5 \times 10^{-3}$ and radius of 0.5 cm, esophagus thickness is 0.3 cm and electrode thickness = 0.008 cm; (c) ratio of the impedance between the absence and presence of bolus as a function of the catheter radius for various esophagus conductivities.

analytical solution using a cylindrical geometry cannot accurately describe the realistic geometries and therefore cannot be used to solve the inverse problem for the conductivities. The variations from the ideal model include esophagus status, bolus contents and the fact that the esophagus is not a perfect cylinder and probes have finite dimensions and conductivities. The finite element technique can be used as an alternative technique to study the effect of various parameters on the impedance. One of the most important design factors of the MII system is the ability to reflect a wide range of changes inside the esophagus as impedance variations. This design is determined by many factors such as catheter configuration and patient status. The catheter configuration includes electrode spacing, electrode length, number of channels and catheter radius. The design curves provided here suggest, for example, that under severe esophageal damage, the inter-electrode spacing needs to be large enough and the catheter radius be as small as possible. The results of this paper can be used for a wide variety of investigations related to the study of multichannel intraluminal impedance.

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